CONDITIONAL CONVERGENCE: AN ANALYSIS OF MUNICIPAL CONVERGENCE CLUBS IN BRAZIL

CONVERGÊNCIA CONDICIONAL: UMA ANÁLISE DOS CLUBES DE CONVERGÊNCIA MUNICIPAIS NO BRASIL

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Maria Cristiane Lopez de Souza

Abstract
This paper analyzes the dynamics of municipal economic growth in Brazil based on the conditional convergence model in clubs. Framed on the theory of conditional convergence, changes in convergence speeds are analyzed under the hypothesis of conditional convergence in clubs. There are two municipal groups of analysis based on regional inequalities: the first, the advanced group with a reduced number of municipalities, high speed of convergence and sensitive to investment shocks; and the second, a backward group with a large number of municipalities, reduced convergence speed and less sensitive to investment shocks, especially concerning human capital. The result suggests that a national policy to encourage investment in reproducible capital with excluding different realities of production may increase economic inequalities among municipalities in countries with robust inequality, such as Brazil.

Keywords: Economic growth, persistent inequalities, Finite Mixtures

Resumo
O trabalho tem o objetivo de analisar a dinâmica de crescimento econômico municipal do Brasil a luz do modelo de convergência condicional em clubes. Inspirado da teoria de convergência condicional, são analisadas as mudanças nas velocidades de convergências sob a hipótese de convergência condicional em clubes. Devido ao quadro persistente de desigualdades regionais, observa-se que existem dois grupos municipais: um grupo avançado, com reduzido número de municípios, alta velocidade de convergência e sensível aos choques de investimentos; e outro grupo atrasado, com grande número de municípios, reduzida velocidade de convergência e menos sensível aos choques de investimentos, especialmente em relação a capital humano. Esse resultado sugere

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that a national policy of stimulating investment in reproducible capital, neglecting the realities of distinct production, can increase economic disparities between municipalities in a country with large economic disparities like Brazil.

**Palavras-Chave:** Crescimento econômico, desigualdades persistentes, Misturas Finitas

**Introduction**

Several local and regional aspects related to human capital, physical infrastructure, and institutions lead to uneven growth in Brazil (GONÇALVES et al., 2011; BARROS, 2011; AZZONI et al., 2000). Studies link inequality of national economic growth to geographic factors (GONDIM et al., 2007), infrastructure (CHEIN et al., 2007; SCHETTINI; AZZONI; 2015), productive efficiency of investments (SCHETTINI; AZZONI; 2018), and technological inequalities (LIMA; URRACA RUIZ, 2020). There is a trend to form economic groups in Brazil, and these groups tend to distance themselves over time (FIGUEIREDO; PORTO JUNIOR, 2015).

The effects of possible economic investment policy shocks on the country's economic growth and municipal inequalities may explain the roots of economic inequalities and their persistence at the municipal level. In Brazil, a national economic policy that neglects specific aspects of some regionalized economic groups may widen the gap of the relative backwardness of municipalities due to regional heterogeneity of the country. Usually, the literature does not address economic inequalities caused by national investment policies that neglect different sensitivity of municipal growth to exogenous shocks of investments in reproducible capital.

In the Brazilian context, two major public investments were made in all regions during the first decade. The first, in physical capital through the Growth Acceleration Plan (PAC) with robust investments in infrastructure. The second, in human capital through the restructuring and expansion programs of Federal Universities (REUNI) and University for All (PROUNI) which expanded the number of federal and private universities by offering grants of scholarships (PAC, 2012; MEC, 2012).

Based on this context, this paper seeks to analyze the dynamics of municipal economic growth in Brazil in light of the conditional convergence model in clubs. It analyzes municipal clubs of economic convergence, regional distribution, the size of inequalities, and the elasticities of reproducible capital.

The paper unfolds in five sections. The next section presents the theoretical framework, including subsections on the technological endogeneity and the extended Solow model, economic convergence, convergence clubs, and an empirical review. Section 3 describes the methodology. Section 4 offers the results and discussion. Section 5 concludes and offers some avenues for future research.

**Theoretical framework**

**Technological endogeneity and the extended Solow model**

The Mankiw model used as a reference in this paper for determining conditional convergence clubs (MANKIW et. al., 1992) constitutes an expansion of Solow's model (1956; 1957), whose hypotheses are of technological, population, and investment exogeneity. The expanded Solow model with human capital is a relevant alternative for empirical studies on economic growth, although the endogeneity of individual choices for human capital formation (LUCAS, 1988), and investments in innovation patented by the almost exclusivity of the capital good (ROMER, 1990). The extended production function is determined by a Cobb-Douglas function, combining with technological level (A), labor (L), physical capital (K), and human capital (H).

\[ Y_i = K_i^a H_i^g (A_i L_i)^{1-a-g} \tag{1} \]

In the same way the accumulation of physical capital \((k_{it})\) depends on the difference between investments \((s_iy_i)\) and depreciation in physical capital \(((n + g + \delta)k_i)\), the human capital \((h_{it})\) also depends on the difference between investments in human capital \((s_iy_i')\) and their depreciation \(((n + g + \delta)h_i)\).
\[ k_i = s_k y_i - (n + g + \delta) k_i \]  
\[ h_i = s_h y_i - (n + g + \delta) h_i \]  

Where \( y, k, \) and \( h \) are defined by \( y = Y/AL; k = K/AL; \) and \( h = H/AL, \) where product, capital, and human capital are per effective unit of work, respectively. The parameters \( s_k \) and \( s_h \) are the rate of investment in physical capital and human capital.

According to Mankiw et al. (1992), if economic growth is endogenous, the return of the reproducible capital stock on production is constant \( (\alpha + \gamma = 1). \) Otherwise, the return is decreasing \( (\alpha + \gamma < 1). \) In addition to the return on reproducible capital being decreasing, rejecting the hypothesis of endogenous growth, the aggregate production of countries is explained by more than 75% when controlling heterogeneity of countries between producers and non-producer's petroleum (MANKIW et al 1992). In equilibrium, the aggregate growth model operates with full use of the factors \((k^*, h^*)\) and responding to exogenous technology shocks \((\ln A(0) + g_i)\).

\[
k^* = \left( \frac{\alpha - \gamma}{n + g + \delta} \right)^{1/\alpha - \gamma} \]
\[
h^* = \left( \frac{\alpha - \gamma}{n + g + \delta} \right)^{1/\alpha - \gamma} \]
\[
\ln\left[\frac{y_i}{L_i}\right] = \ln A(0) + g_i - \frac{\alpha + \gamma}{1 - \alpha - \gamma}\ln(n + g + \delta) + \frac{\alpha}{1 - \alpha - \gamma}\ln(s_k) + \frac{\gamma}{1 - \alpha - \gamma}\ln(s_h) \]  

**Economic convergence**

Barro and Sala-i-Martin (1992) and Mankiw et al. (1992) develop the conditional convergence analysis based on the expanded Solow model with human capital, in a steady-state. The rate of economic growth decreases as the capital stock approaches steady-state (MANKIW et al, 1992). Countries converge conditionally to their steady-state as economies differ in steady-state. The model \( \beta \)-convergence is developed to analyze the speed of economic convergence in the countries. Based on this model, it is feasible not only the conditional convergence to the long-term equilibrium of economies but also to estimate the speed which they reduce their inequalities (BARRO; SALA-I-MARTIN, 1992).

**Convergence clubs**

Convergence clubs capture trajectories of economic groups with distinct long-term balances through non-parametric models of dynamic distribution of aggregate income. The probability distribution of *per capita* income over time shows that economic groups tend to form two modal peaks in the long run. These peaks in the probability distribution of *per capita* income represent economic groups with a different modal value of *per capita* income. The higher modal *per capita* income group is the advanced economic club, and the lower modal *per capita* income group is the backward club. Economic groups are the convergence clubs based on the theory of decreasing return on capital stock (Quah, 1996; 1997).

**Empirical Review**

Most studies show that national economic inequality tends to form convergence clubs where there is a reduction in inequalities within economic groups and an increase in inequalities among groups. There is a trend towards the formation of convergence clubs, with polarization between rich and poor economic groups (JÚNIOR; RIBEIRO, 2003). The concentration of wealth in the South may further increase relative backwardness of the North and Northeast regions in the absence of exogenous shocks favorable to the distribution of income among Brazilian states and regions (JÚNIOR; RIBEIRO, 2003).

According to Schettini and Azzoni (2015), the uneven distribution of the country's industrial infrastructure is the main determinant of regional inequality. Road, urban, telephone, and internet infrastructure have had a positive impact on production efficiency, increasing the industrial product. The efficiency of investments in the present time indicates the fate of investments in infrastructure. Therefore, there is a tendency for income concentration in the country to favor more efficient...
regions. Without public intervention, it is necessary new sources of stimulus for private investment due to budgetary limitations (SCHETTINI; AZZONI, 2018).

Municipal groups form convergence clubs because of their different capacities for economic growth. It is determined by differences in physical, human capital, productive infrastructure, and technological inequalities. Technological drivers are the only factors capable of changing the development status of municipalities in backward economic clubs, although reproducible capital stock being relevant to determine municipal economic growth. The main technological drivers are the choices of investment in research and development (R&D), institutional quality, technology transfer, and investments in technological infrastructure (LIMA; URRACA RUIZ, 2020).

Methodology

Conditional convergence consists of the hypothesis that an economy’s growth rate is inversely related to the proximity of its steady-state. This conclusion implies that economies converge on their long-term balance when the return on reproducible capital is decreasing (MANKIW et al., 1992). The conditional convergence hypotheses of Brazilian municipalities are tested between 2000 and 2010 using the Solow equilibrium model and the Solow model expanded with human capital. The hypotheses are tested using the Barro and Sala-i-Martin (1992) β-convergence analysis model. The model is estimated using the OLS estimators for conditional convergence and Finite Mixes for conditional convergence clubs.

The theoretical model of conditional convergence

The specification of the conditional convergence equation considers at the same time the initial per capita income \((y_0)\) in allusion to production with unemployment, and the steady equilibrium per capita income of the economy \((y^*)\). The parameter of economic convergence becomes more realistic as the specification of the aggregate production function becomes more feasible. By adding and subtracting \(\ln(y_0)\) in the function of the initial income \((y_0)\), the equation (7) measures the relationship between the economic growth rate \((\ln(y_i) − \ln(y_0))\) conditioned to the steady-state \((y^*)\).

\[
\ln(y_i) = (1 − e^{-\lambda i}) \ln(y^*) + e^{-\lambda i} \ln(y_0) \quad (7)
\]

\[
\ln(y_i) − \ln(y_0) = (1 − e^{-\lambda i}) \ln(y^*) − (1 − e^{-\lambda i}) \ln(y_0) \quad (8)
\]

The substitution of parameters for the equilibrium capital stock of the Solow model and the physical and human capital stocks of the extended Solow model in convergence equations build the conditional convergence equations. It considers two distinct equilibrium points in the long run. The conditional convergence with the Solow equilibrium is given as follows:

\[
\ln(y_{it}) - \ln(y_{oit}) = (1 - e^{-\lambda i}) \frac{\alpha}{1-\alpha-\gamma} \ln(s_{kit}) - (1 - e^{-\lambda i}) \frac{\alpha+\gamma}{1-\alpha-\gamma} \ln(n + g + \delta) - (1 - e^{-\lambda i}) \ln(y_{oit}) \quad (9)
\]

The conditional convergence with extended Solow equilibrium:

\[
\ln(y_{it}) - \ln(y_{oit}) = (1 - e^{-\lambda i}) \frac{\alpha}{1-\alpha-\gamma} \ln(s_{kit}) + (1 - e^{-\lambda i}) \frac{\gamma}{1-\alpha-\gamma} \ln(s_{hit}) - (1 - e^{-\lambda i}) \frac{\alpha+\gamma}{1-\alpha-\gamma} \ln(n + g + \delta) - (1 - e^{-\lambda i}) \ln(y_{oit}) \quad (10)
\]

An empirical model of conditional convergence

This paper uses the conditional convergence model to the simple Solow equilibrium, and Solow expanded with human capital. In these models, economic convergence is conditioned to equilibrium when reproducible capital is only physical capital \((s_{ki})\) and reproducible capital is the combination of physical and human capital \((s_{ki}, s_{hi})\). The conditional convergence to Solow equilibrium (equation 11), and extended Solow equilibrium with human capital (equation 12):

\[
\Delta \ln(y_i) = \beta_1 \ln(y_{2000i}) + \alpha_1 \ln(s_{ki}) + \alpha_2 \ln(n + g + \delta)_i + \alpha_3 + \alpha_{n-1} d_{n-1} + \varepsilon_i \quad (11)
\]

\[
\Delta \ln(y_{it}) = \beta_1 \ln(y_{2000it}) + \alpha_1 \ln(s_{ki}) + \alpha_2 \ln(s_{hit}) + \alpha_3 \ln(n + g + \delta)_i + \alpha_4 + \alpha_{n-1} d_{n-1} + \varepsilon_i \quad (12)
\]

Where:

\(\Delta \ln y_i\) is the growth rate of per capita income of municipalities between 2000 and 2010;

\(\ln y_{2000i}\) is the municipalities’ initial income;

\(\ln s_{ki}\) is the average physical capital stock;
\[ \ln s_{hi} \] is the average human capital stock;
\[ \ln (n+g+\delta) \] is the depreciation term, given by the population growth rate;
\[ d_{n-1} \] are dummies of the country’s regions to control regional heterogeneity;
\[ \varepsilon_i \] is an unobserved random disorder.

The convergence speed (\( \lambda \)) of steady-state economies applies the estimated parameter \( \beta \) of the empirical equation. Its construction derives from the estimated parameter \( \beta = 1 - e^{-\lambda t} \) of the convergence equation. The convergence speed (\( \lambda \)) is an inverse function of estimated \( \beta - convergence \). The half-life (\( \vartheta \)) applies the parameter \( \lambda \), whose value is an inverse function of convergence speed. The half-life (\( \vartheta \)) measures the time to reduce half of the inequality between the most advanced and backward economies\(^4\) (BARRO; SALA-I-MARTIN, 1992).

**An empirical model of conditional convergence clubs**

**Convergence clubs grouped by Kernel density function**

The kernel function of probability density can identify the dynamic distribution of the municipal convergence clubs between 2000 and 2010. The distribution of the municipal convergence clubs consists of a generalization of the histogram given by an alternative function of weights \( f(y) \). Where \( k(y) \) is the kernel function, \( N \) the number of observations, and \( h \) the smoothing parameter (CATELA; GONÇALVES, 2009). Convergence presupposes a change in the distribution of per capita income (\( y \)) of municipalities over time towards more equitable levels of homogeneous average, tapering of the distribution, and approximation of the tails of the two-tailed distribution.

\[
\hat{f}(y_0) = \frac{1}{Nh} \sum^{N}_{i=1} k \left( \frac{y_i - y_0}{h} \right)
\]

(13)

The density functions illustrate exploratory data analysis according to multimodality, asymmetry, and kurtosis. The simplicity, absence of bias, and consistency justify the use of this estimator in the characterization of variables. The kernel function is a non-parametric statistic, so it does not depend on the prior knowledge of the data distribution (LUCAMBIO, 2008). The logarithm in per capita income variables of probability distribution correct data distortions and outliers (BIANCHI, 1997). The formation of modal peaks in probability distribution presupposes the formation of convergence clubs in Brazilian municipalities, but it does not determine elasticities of production factors of the aggregate function of grouped municipalities.

**Convergence clubs estimated by the Finite Mixture Method**

This estimator determines the convergence clubs by the similarity of the probability distribution of growth rate conditioned to determinants of economic growth. The growth rate with conditional distribution at different means and variances and the different parameters estimated for each convergence club justify its use (OWEN et. Al., 2009). The estimation does not order the municipalities in advanced and backward groups, it only identifies the differences between the elasticities of growth determinants of each club (CAMERON; TRIVEDI, 2005). To order clubs, it is important saving the categorical latent variable based on the most likely latent class association. This procedure can separate economic groups according to their latent class by the level of growth, initial income, and factors of production. The results can be confirmed by the later probability of the latent class (DEB, 2008).

The Finite Mixtures Estimator assumes the density of \( y \) is a linear combination of \( m \) different densities given by \( f(y/\theta_j) \) with \( j=1, ..., m \) densities. The estimated components capture the modal

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\(^4\) Convergence equation: \[ \ln(y_j) - \ln(y_0) = (1 - e^{-\lambda t}) \ln(y_i) - (1 - e^{-\lambda t}) \ln(y_0) \]

Convergence parameter: \[ -\beta = -(1 - e^{-\lambda t}) \] in which \( 0 < \beta < 1 \)

Adding zero: \[ 1 - 1 - \beta = -(1 - e^{-\lambda t}) \; 1 + 1 - (1 - e^{-\lambda t}) = 1 - \beta = e^{-\lambda t} \]

Taking the ln: \[ -\lambda t \ln(\varepsilon) = \ln(1 - \beta) \]

Converging speed: \[ \lambda = -\frac{\ln(1 - \beta)}{t \ln(\varepsilon)} \]

Half-life: \[ \vartheta = \frac{\ln(2)}{\lambda} \]
values of the probability density function, grouping the population of data into subpopulations \( \pi_j \)
called convergence clubs, whose \( \sum_{j=1}^{m} \pi_j = 1 \) (CAMERON; TRIVEDI, 2005). The estimated empirical equation is described as follows:

\[
f(\Delta \ln y_i / \theta, \pi) = \sum_{j=1}^{m} \pi_j f_i (\Delta \ln y_i / \ln y_{2000i}, \ln s_{ki}, \ln s_{hi}, d_{n-1}i) + \varepsilon_i
\]

(15)

Where:

\( \Delta \ln y_i \) is per capita income in period \( t \);
\( \theta = \ln y_{2000i}, \ln s_{ki}, \ln s_{hi}, d_{n-1}i \) correspond to the vector of explanatory variables, determined by initial income \( (\gamma_{2000i}) \), physical and human capital \( (s_{ki}, s_{hi}) \), and regional dummies \( (d_{n-1}) \) to control heterogeneities of geographical regions of Brazil;

\( (n+g+\delta) \) are the model’s depreciation parameters given by population growth;
\( \varepsilon_i \) is a random disturbance not observed by the model.

Presentation of variables and data sources

The municipal GDP per capita in the years 2000 and 2010 determine the municipal per capita income \( (\gamma_i) \) used to estimate the kernel density function, municipal economic growth rate \( (\Delta \ln y_i) \) used in the conditional convergence models, and the variables of initial income \( (\gamma_{2000i}) \). These variables are available in the IBGE and IPEA database. The ratio between the average value added to sectoral production weighted by the capital participation in each sector and the aggregate municipal GDP determines the physical capital investment proxy \( (s_{ki}) \) (SANTOS; SPOLADOR, 2018). Following Siqueira (2019), the average of the sectorial GDP weighted by the participation of capital in production determines the capital income in municipal aggregate production. Data on value-added of sectoral production are available in the IBGE database, and the participation of sectorial capital is available in Santos and Spolador (2018).

The number of people with completed secondary education linked to formal labor market activity in the total municipal population determines the human capital investment rate proxy \( (s_{hi}) \). According to Gonçalves et al. (2011), this proxy is a rate of investment in human capital. Data on secondary education is available in the Ministry of Labor’s Annual Social Information Database (RAIS), and the municipal population is available in the IPEA database. The difference in natural logarithm (\( \ln \)) of the population estimate of municipalities between 2000 and 2010 determines population growth \( (n_i) \). Following the analysis of conditional convergence in two periods of Mankiw et al. (1992), the physical and human capital proxies are averages for 2000 and 2010. All monetary variables are deflated to 2000 using the General Price Index - Internal Availability (IGP-DI) of Fundação Getúlio Vargas.

Frame 1: variables and data source

<table>
<thead>
<tr>
<th>Variables</th>
<th>Definition</th>
<th>Years</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \ln y_i )</td>
<td>The growth rate of per capita income at constant 2000 prices.</td>
<td>2000-2010</td>
<td>IBGE</td>
</tr>
<tr>
<td>( \ln y_{2000} )</td>
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<td>2000-2010</td>
<td>IBGE</td>
</tr>
<tr>
<td>( \ln s_{ki} )</td>
<td>Average 2000-2010 of the ratios between investment weighted by the participation of sectoral capital and municipal GDP at constant 2000 prices.</td>
<td>2000-2010</td>
<td>IBGE (SANTOS e SPOLADOR, 2018)</td>
</tr>
<tr>
<td>( \ln s_{hi} )</td>
<td>Average 2000-2010 of participation of the number of people with complete secondary education in the formal job market of the total population of the municipality.</td>
<td>2000-2010</td>
<td>RAIS</td>
</tr>
</tbody>
</table>

5 Brazilian Institute of Geography and Statistics (IBGE). In portuguese: Instituto Brasileiro de Geografia e Estatística (IBGE).
6 Institute of Applied Economic Research (IPEA). In portuguese: Instituto de Pesquisa Econômica Aplicada (IPEA).
The depreciation parameter is given by population growth. The calculation is determined by the difference in the ln of the municipal population between 2000 and 2010.

<table>
<thead>
<tr>
<th>Variables</th>
<th>N. Obs.</th>
<th>Mean</th>
<th>SD</th>
<th>CV (%)</th>
<th>Min.</th>
<th>Max.</th>
</tr>
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<tbody>
<tr>
<td>$\Delta \ln y_i$</td>
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<tr>
<td>$(n + g + \delta)_i$</td>
<td>4862</td>
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Note: monetary variables were deflated at 2000 prices using the IGP-DI FGV.

Results and discussion

Conditional convergence

The economy's level of socio-cultural development reflects population growth. A high Coefficient of Variation (CV) reflects the different realities of Brazilian municipalities. Backward economies tend to have high population growth $(n + g + \delta)_i$ and low economic growth. In advanced economies, the dynamic is low population growth and high economic growth (MANKIW et al., 1992; GALOR, 2007). The stock of human capital is considered as one of the most relevant factors for aggregate economic growth to explain regional economic inequalities in Brazil (BARROS, 2011). It reflects the inequality of Brazilian human capital $(s_{hi})$, whose coefficient of variation (CV) is the third-largest among other variables in the estimated model with approximately 72% of dispersion around the national average. Also, the model's variables are strongly associated with the dispersion of almost 31% of the economic growth rate $(\Delta \ln y_i)$. The rate of investment in physical capital $(s_{ki})$ exhibits the lowest national dispersion coefficient. The human capital is more heterogeneous in Brazil due to a lower average than physical capital and a higher dispersion rate. Therefore, further discussion is necessary about human capital because of its importance in the country's aggregate production (Table 1).

The conditional convergence model uses the balance of the simple Solow model for Brazilian municipalities and reflects the bias discussed by Mankiw et al. (1992). According to Mankiw et al. (1992), it reflects the absence of human capital in the aggregate production function which generates a specification bias in the estimation of the physical capital investment rate parameter $(\ln s_{ki})$. The average rate of capital investment harms municipal economic growth. Even though, the model presents evidence of conditional convergence due to the negative parameter of the initial income variable $(\ln y_{2000})$. The statistical significance of regional dummies about Northeast and the depreciation parameter $(\ln (n + g + \delta)_i)$, demonstrate the theoretical condition of the effect of population growth on economic growth and the relative backwardness of Northeast in other regions in the country (Table 2).

The conditional convergence model of the Solow expanded with human capital tends to correct distortions on the estimated parameters compared to the simple model. The positive effect of the average investment rate of physical capital increases by 1% in investment and presents an average effect of 0.53% on municipal economic growth. The average parameter $\beta$-convergence advances from a negative 0.12 in the simple model to a negative greater than 0.26 in the extended

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Table 1: Descriptive statistics of the variables in the Solow model with human capital

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Note: CV is coefficient of variation; SD is standard deviation.

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7 Economies would move from the Malthusian stage with high population growth and low economic growth to the stage of sustainable development with low population growth and great economic growth in the stages of economic development of Galor (2007). This behavioral change would lead to the formation of smaller families and the generation of technological innovations with the formation of human capital and the expansion of gross domestic product per capita.
model, increasing the speed of convergence of the municipalities about their steady-state. Econometric specifications that imply an increase in the speed of convergence consist of more consistent models in the analysis of economic convergence (BARRO; SALA-i-MARTIN, 1992). The average human capital investment rate parameter has a positive average effect of 0.24% for each 1% incremental change in the human capital investment rate. Also, the inclusion of human capital in the convergence specification of the extended model increases the average ability to explain the regression by 9.17%. The Coefficient of determination (R2) increases from 0.1754 in the simple model to 0.2671 in the extended model (Table 2).

Table 2: Conditional convergence models

<table>
<thead>
<tr>
<th>Variables</th>
<th>Convergence - Simple</th>
<th>Convergence - Extended</th>
</tr>
</thead>
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<tr>
<td>( \ln y_{2000} )</td>
<td>-0.1273723*** (0.0103357)</td>
<td>-0.2676788*** (0.0148446)</td>
</tr>
<tr>
<td>( \ln s_{hi} )</td>
<td>-0.1918488*** (0.0721355)</td>
<td>0.5322327 *** (0.088225)</td>
</tr>
<tr>
<td>( \ln s_{hi} )</td>
<td>0.2467605*** (0.0158343)</td>
<td>0.2467605*** (0.0158343)</td>
</tr>
<tr>
<td>( \ln (n + g + \delta)_{i} )</td>
<td>-0.4611243*** (0.0269255)</td>
<td>-0.4103974*** (0.0255367)</td>
</tr>
<tr>
<td>North</td>
<td>0.1471124*** (0.0197083)</td>
<td>0.1377117*** (0.0179364)</td>
</tr>
<tr>
<td>Southeast</td>
<td>0.0509053*** (0.0509053)</td>
<td>0.0262122** (0.0128043)</td>
</tr>
<tr>
<td>South</td>
<td>0.1003998*** (0.0164551)</td>
<td>0.0458481*** (0.0157501)</td>
</tr>
<tr>
<td>Midwest</td>
<td>0.1883142*** (0.0206812)</td>
<td>0.1444107*** (0.0188586)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.9413439*** (0.0638016)</td>
<td>2.059213*** (0.1000365)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.1754</td>
<td>0.2671</td>
</tr>
</tbody>
</table>

Source: the authors  
Note: *** significant at 1%, ** significant at 5%; the values in parentheses are the standard error estimates of the estimated parameters.

Conditional Convergence Clubs

The different effects of production factors on municipal economic growth in the different convergence clubs can influence the formulation of localized policies to control inequalities. The identification of modal peaks in the distribution is evidence of the formation of convergence clubs, as suggested by Quah (1996) in the “twin peaks” hypothesis. In this way, Figure 1 shows the existence of two clearly defined municipal groups in the years 2000 and 2010. The only difference between the kernel density functions is that in 2010 the clubs are even tighter, demonstrating less variance around modal values.
The estimation of convergence clubs is useful to understand the size of municipal economic groups, their regional distribution, the size of inequalities, and the level of sensitivity of economic growth to factors of production. These aspects are discussed throughout the analyzes of Tables 4 and 5, and Figure 2. To determine the number of municipal convergence clubs, two regressions were performed with two and three components using the Finite Mixture Method (Table 4). The method generates clubs based on the similarity of the probability distribution of production factors (OWEN et al., 2009). The Kernel probability density function points to the existence of two economic groups; so, there is a chance to have only two convergence clubs.

The parameters generated along with the estimates guide the choice of the best regression: latent class posterior average probability; Akaike and Bayesian information criteria; and Entropy coefficient (see table 3). The latent class posterior average probability estimates the probability the municipality belongs to the latent class and is outside the group of its estimated component. The greater the probabilities of the main diagonal of the regressions with 2 and 3 Latent Classes, the more adjusted is the regression because it indicates the probability the municipality belongs to the estimated latent class. The Akaike and Bayesian information criteria suggest the choice of models estimated with the lowest AIC and BIC values. The Entropy coefficient, which varies from 0 to 1, indicates that the most adjusted model has the highest coefficient.

Based on this set of evidence, two-component regression is chosen. As the mean posterior probability of the two-component regression shows the lowest probabilities were the main diagonal indicating a low probability the municipality does not belong to the estimated latent class, the entropy coefficient is higher than in the three-component regression. The change in the value of criteria between regressions with two and three components is small (Table 3) despite the information criteria AIC and BIC are contrary to this choice. What is more, the kernel density function of municipal GDP per capita suggests the existence of only two convergence clubs (Figure 1). In allusion to two convergence clubs, the estimation with two latent classes has statistical significance and sign of the estimated parameters according to theory, unlike the regression with three latent classes (Table 4).
Table 3: Adjustment of estimates based on the posterior mean probability of the latent classes of the equations, AIC and BIC information criteria, and Entropy

<table>
<thead>
<tr>
<th>Mean</th>
<th>Regression with two components</th>
<th>Regression with three components</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CL1</td>
<td>CL2</td>
</tr>
<tr>
<td>P1</td>
<td>0.904</td>
<td>0.096</td>
</tr>
<tr>
<td>P2</td>
<td>0.143</td>
<td>0.857</td>
</tr>
<tr>
<td>P3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Akaike (AIC)</td>
<td>-1138.243</td>
<td></td>
</tr>
<tr>
<td>Bayesian (BIC)</td>
<td>-1001.969</td>
<td></td>
</tr>
<tr>
<td>Entropy</td>
<td>0.580</td>
<td></td>
</tr>
</tbody>
</table>

Source: the authors

Based on the estimation of two convergence clubs, on average, the most homogeneous economic group of Latent Class 2 converges at a speed almost twice as fast as the most heterogeneous group Latent Class 1. The $\beta$-convergence parameter increases its negative value from 0.23 to 0.43 of Latent Class 1 to Latent Class 2. The municipalities of North, South, and Center-West regions grew on average at a rate higher than Northeast, suggesting an expansion of inequalities in these regions compared to Northeast in Latent Class 1 of the convergence club. The economic growth rate of municipalities in North, Southeast, and Midwest regions is on average above the growth in Northeast in Latent Class 2 convergence club. In both clubs, there is a relative delay in municipalities of Northeast compared to others (Table 3).

The elasticities of physical capital and human capital proxies increase strongly from Latent Class 1 club to Latent Class 2 club. The elasticity of human capital has the greatest increase; from 0.15 of the latent Class 1 to 0.54 in the Latent Class 2. The results suggest a shock of 10% in national investment in human capital which generates an approximate average increase of 15% in economic growth in municipalities of Latent Class 1 convergence club and an approximate average increase of 54% in municipal growth of Latent Class 2 convergence club. For the investments in physical capital, the same 10% would lead to the average economic growth of approximately 39% in municipalities of Latent Class 1 convergence club and 56% in municipalities in Latent Class 2 convergence club (Table 3).

The results suggest investments in reproducible capital in Latent Class 2 convergence club have a greater effect on economic growth compared to Latent Class 1 club. An economic policy that promotes an increase of 1% in the reproducible capital of all municipalities in the country in two convergence clubs inevitably creates more inequalities between economic groups. Therefore, it is crucial to know the differences in the sensitivity of production factors on municipal economic growth between convergence clubs. A policy aimed at correcting economic inequalities should include differences in regional economic infrastructure favoring the club with the least elasticity, increasing investments where its effects generate lesser results. Greater investment in human capital can increase the productivity of factors, breaking the status of relative economic backwardness in economies (OWEN et al., 2009; BARROS, 2011).
### Table 3: Convergence Clubs estimated by the Finite Mixture Method

<table>
<thead>
<tr>
<th>Variables</th>
<th>Two-component regression</th>
<th>Three-component regression</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Latent class 1</td>
<td>Latent class 2</td>
</tr>
<tr>
<td>ln $y_{2000i}$</td>
<td>- 0.238461*** (0.0107646)</td>
<td>0.4304951*** (0.062959)</td>
</tr>
<tr>
<td>ln $s_{ki}$</td>
<td>0.3971597*** (0.0700787)</td>
<td>0.5605053*** (0.2593641)</td>
</tr>
<tr>
<td>ln $s_{hi}$</td>
<td>0.1512511*** (0.0110295)</td>
<td>0.5476176*** (0.0753299)</td>
</tr>
<tr>
<td>ln($n + g + \delta$)$_i$</td>
<td>-0.497486*** (0.0220892)</td>
<td>-0.1811428* (0.0955653)</td>
</tr>
<tr>
<td>North</td>
<td>0.0823661*** (0.0202236)</td>
<td>0.327018*** (0.0574775)</td>
</tr>
<tr>
<td>Southeast</td>
<td>0.0072588 (0.0104946)</td>
<td>0.1921387*** (0.0655703)</td>
</tr>
<tr>
<td>South</td>
<td>0.0865757*** (0.0136355)</td>
<td>0.0023243 (0.0664343)</td>
</tr>
<tr>
<td>Midwest</td>
<td>0.1491969*** (0.0152148)</td>
<td>0.2091092*** (0.0799036)</td>
</tr>
<tr>
<td>Constant</td>
<td>1.760703*** (0.0779262)</td>
<td>2.714724*** (0.3205567)</td>
</tr>
</tbody>
</table>

Source: the authors

Note: *** significant at 1%, ** significant at 5%; * significant at 10%; the values in parentheses are the standard error estimates of the estimated parameters.

The average values of the variables of the model estimated by Latent Class in Table 4 present the average of economic growth, initial income, and human capital are higher in the municipalities of Latent Class 2 compared to Latent Class 1. The municipalities of Latent Class 2 constitute the advanced convergence club and Latent Class 1 the backward convergence club. The regional distribution of the convergence clubs shows that Southeast, Midwest, and North have most of their respective municipalities in the advanced convergence club, although 57% of municipalities in the advanced club are from South and Southeast regions (Figure 2b). The North and Center-West Regions jump from 8% and 6% of the municipalities of the backward club to 13% of the advanced municipalities. The Northeast Region is the most accentuated case, with 31% of the municipalities in the backward club, and 18% of the municipalities in the advanced club although great inequality in the richest regions of the country, such as Southeast and South of Brazil. The Northeast is the only region whose absolute majority of municipalities are tied to a backward convergence club with low economic growth and low rates of investment in reproducible capital (Figure 2).
Table 4: Descriptive statistics of the variables by estimated component

<table>
<thead>
<tr>
<th>Variables</th>
<th>Latent class 1</th>
<th>Latent class 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SD</td>
</tr>
<tr>
<td>Δlnyi</td>
<td>0.937506</td>
<td>0.220728</td>
</tr>
<tr>
<td>lnσki</td>
<td>0.416746</td>
<td>0.032465</td>
</tr>
<tr>
<td>lns_hki</td>
<td>0.368648</td>
<td>0.260188</td>
</tr>
<tr>
<td>ln(n + g + δ)_i</td>
<td>0.100241</td>
<td>0.1735</td>
</tr>
</tbody>
</table>

N. Obs. 4446 416

Source: the authors

Figure 2: Regional convergence clubs by backward and advanced (a) economies (b)

The convergence speed (λ) and half-life (υ) parameters are indexed to the estimated parameter β-convergence (Table 5). Therefore, the time needed to reduce half of the inequalities between the most backward and most advanced economies (υ) would be approximately 73 years in the simple Solow model. The time falls to 32 years with the specification of the extended Solow model. The time to reduce inequalities drops to 17 years in the advanced convergence club in localities with a greater speed of convergence (λ). The convergence time is very similar to the conditional convergence model of the extended Solow model in the delayed convergence club. The average result suggests other convergence sub clubs within the delayed convergence club because of different elasticities between municipal groups. Exhaustive estimates of convergence sub clubs are unnecessary given different elasticities among municipal groups.

Table 5: Convergence speed and half-life

<table>
<thead>
<tr>
<th>Reducing inequalities</th>
<th>Conditional convergence</th>
<th>Convergence clubs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simple Solow</td>
<td>Expanded Solow</td>
</tr>
<tr>
<td>β</td>
<td>0.1273723</td>
<td>-0.2676788</td>
</tr>
<tr>
<td>λ</td>
<td>0.0136246</td>
<td>0.0311536</td>
</tr>
<tr>
<td>υ</td>
<td>73.396502</td>
<td>32.099012</td>
</tr>
</tbody>
</table>

Source: the authors

Conclusion

Conditional convergence municipal clubs exhibit different elasticities of production factors over economic growth. Advanced club municipalities are much more sensitive of changing rate investments of physical and human capital than backward municipalities. This result suggests that a national policy to encourage investment in reproducible capital regardless of the specificities of these groups may increase economic inequalities between two convergence clubs. Therefore, a policy of reducing inequalities should favor the backward convergence club more so as not to increase economic inequalities.

The descriptive statistics of the variables in the estimated model point to an average advantage of the economic growth rate, initial income per capita, and investment rate for the Latent Class 2 municipal club compared to the Latent Class 1 club. There is still a high speed of convergence with a small number of municipalities highly sensitive to exogenous investment shocks, that is, the advanced convergence club. In the municipal group of the Latent Class 1, an opposite situation is
verified. In this municipal club, there is a low speed of convergence, an exceedingly higher number of municipalities, and low sensitivity to exogenous investment shocks, that is, the delayed convergence club. The regions of Southeast, Southwest, and North, have most of their municipalities in the advanced club, while Northeast maintains an absolute advantage of its municipalities in the backward convergence club.

New researches on convergence clubs can explore intra-regional heterogeneity to develop policies tailored to each of the national realities. The effects of innovation can also be verified through the technological diffusion of investments in R&D and the absorption of technologies. Also, studies can be conducted in other countries with strong inequalities such as Brazil.

References


